ps

ntity

**Choose the correct vocabulary term to complete each sentence.**

1. You can find missing measures of any triangle by using the __?__ if you know the measures of two angles and a side. **Law of Sines**
2. The six ratios of the lengths of the sides of a right triangle are known as the __?__.
3. If you know the measures of two sides and the angle between them, you can use the __?__ to find missing parts of any triangle. **Law of Cosines**
4. A trigonometric equation that is true for all values except those for which the expressions on either side of the equal sign are undefined is a __?__.
5. The __?__ can be used to find missing measures of any triangle when you know two sides and the angle opposite one of them. **Law of Sines**

## oncepts

tric

A **trigonometric identity** is a trigonometric equation that is true for all values except those for which the expressions on either side of the equal sign are undefined.

| Reciprocal Identities | Tangent and Cotangent Identities | Pythagorean Identities |
|---|---|---|
| $\csc\theta = \frac{1}{\sin\theta}$ | $\tan\theta = \frac{\sin\theta}{\cos\theta}$ | $\cos^2\theta + \sin^2\theta = 1$ |
| $\sec\theta = \frac{1}{\cos\theta}$ | $\cot\theta = \frac{\cos\theta}{\sin\theta}$ | $1 + \tan^2\theta = \sec^2\theta$ |
| $\cot\theta = \frac{1}{\tan\theta}$ | | $1 + \cot^2\theta = \csc^2\theta$ |

**Verify each identity.** **6–7. See margin.**

6. $\sin\theta\tan\theta = \frac{1}{\cos\theta} - \cos\theta$

7. $\cos^2\theta\cot^2\theta = \cot^2\theta - \cos^2\theta$

**Simplify each trigonometric expression.**

8. $1 - \sin^2\theta$ **$\cos^2\theta$**

9. $\frac{\cos\theta}{\sin\theta\cot\theta}$ **1**

10. $\csc^2\theta - \cot^2\theta$ **1**

of
ons

ric

The function $\cos^{-1} x$ is the inverse of $\cos\theta$ with the restricted domain $0 \le \theta \le \pi$. The function $\sin^{-1} x$ has domain $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, and $\tan^{-1} x$ has domain $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.

**Use a unit circle and 30°-60°-90° triangles to find the value in degrees of each expression.**

11. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ **−60°**

12. $\tan^{-1}\sqrt{3}$ **60°**

13. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ **−30°**

14. $\cos^{-1}\frac{\sqrt{3}}{2}$ **30°**

15. $0.34 + 2\pi n, 2.80 + 2\pi n$
16. $-1.11 + 2\pi n, 2.03 + 2\pi n$
17. $2.27 + 2\pi n, 4.02 + 2\pi n$
18. $0.20 + 2\pi n, 6.08 + 2\pi n$

**Use a calculator to find the value in radians of each expression.**

**15.** $\sin^{-1} 0.33$ **16.** $\tan^{-1}(-2)$ **17.** $\cos^{-1}(-0.64)$ **18.** $\cos^{-1} 0.98$

**Solve each equation for $0 \le \theta < 2\pi$.**

**19.** $2\cos\theta = 1$ $\frac{\pi}{3}, \frac{5\pi}{3}$

**20.** $\sqrt{3}\tan\theta = 1$ $\frac{\pi}{6}, \frac{7\pi}{6}$

**21.** $\sin\theta = \sin^2\theta$ $0, \frac{\pi}{2}, \pi$

---

### 14-3 Objectives

- To find lengths of sides in a right triangle (p. 792)
- To find measures of angles in a right triangle (p. 795)

The six different ratios of the sides of a right triangle are known as the **trigonometric ratios for a right triangle.** These ratios depend on the size of the acute angles in the right triangle.

In a right triangle $ABC$ that has an acute $\angle A$ and right $\angle C$, the ratios are defined as follows.

$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{a}{c} \qquad \csc A = \frac{\text{length of hypotenuse}}{\text{length of leg opposite } \angle A} = \frac{c}{a}$$

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{b}{c} \qquad \sec A = \frac{\text{length of hypotenuse}}{\text{length of leg adjacent to } \angle A} = \frac{c}{b}$$

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{a}{b} \qquad \cot A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of leg opposite } \angle A} = \frac{b}{a}$$

**In $\triangle ABC$, $\angle B$ is a right angle, $AB = 30$, and $\sec A = \frac{5}{3}$. Find each value in fraction and in decimal form.**

**22.** $\cos A$ $\frac{3}{5}$, 0.6 **23.** $\sin A$ $\frac{4}{5}$, 0.8 **24.** $\tan C$ $\frac{3}{4}$, 0.75

**In $\triangle GHI$, $\angle H$ is a right angle. Find the remaining sides and angles. Round your answers to the nearest tenth.**

**25.** $g = 3, i = 9$ $3\sqrt{10}$, 18.4°, 71.6°

**26.** $g = 12, h = 20$ 16, 36.9°, 53.1°

**27.** $h = 55, i = 40$ 37.7, 43.3°, 46.7°

---

### 14-4 Objectives

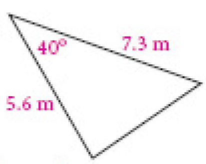
- To find the area of any triangle and to use the Law of Sines (p. 801)

You can find missing measures of any triangle if you know the measures of two angles and any side or if you know the measures of two sides and the angle opposite one of them by using the **Law of Sines.**

The Law of Sines states that for $\triangle ABC$, if $a$, $b$, and $c$ represent the lengths of the sides opposite $\angle A$, $\angle B$, and $\angle C$, respectively, then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

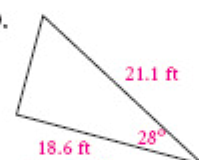
**Find the area of each triangle. Round your answers to the nearest hundredth.**

**28.**



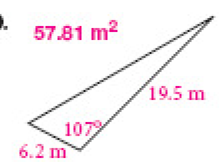

13.14 m²

**29.**




92.12 ft²

**30.** 57.81 m²




**31.** In $\triangle LMN$, $m\angle L = 67°$, $m\angle N = 24°$, and $MN = 16$ in. Find $LM$ to the nearest tenth. 7.1 in.

**32.** In $\triangle XYZ$, $m\angle Z = 34°$, $x = 61$ cm, and $z = 42$ cm. Find $m\angle X$ to the nearest tenth. 54.3° or 125.7°

Cosines to find the area of any triangle (p. 808)

$a^2 = b^2 + c^2 - 2bc\cos A \quad b^2 = a^2 + c^2 - 2ac\cos B \quad c^2 = a^2 + b^2 - 2ab\cos C$

33. 21.4 ft

34. 43.9°

**33.** In $\triangle ABC$, $m\angle B = 45°$, $a = 24$ ft, and $c = 30$ ft. Find $b$ to the nearest tenth.

**34.** In $\triangle DEF$, $d = 25$ in., $e = 28$ in., and $f = 20$ in. Find $m\angle F$ to the nearest tenth.

**35.** In $\triangle GHI$, $h = 8$, $i = 12$, and $m\angle G = 96°$. Find $m\angle I$ to the nearest tenth. 52.2°

---

### 4-6 Objectives

- To verify angle identities (p. 814)
- To verify sum and difference identities (p. 816)

**Angle identities** are used to solve trigonometric equations.

**Negative angle identities**

$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta \qquad \tan(-\theta) = -\tan\theta$

**Cofunction identities**

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta \qquad \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$

**Angle difference identities**

$\sin(A - B) = \sin A\cos B - \cos A\sin B \qquad \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

$\cos(A - B) = \cos A\cos B + \sin A\sin B$

**Angle sum identities**

$\sin(A + B) = \sin A\cos B + \cos A\sin B \qquad \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

**Verify each identity.** 36–37. See margin.

**36.** $\cos\left(\theta + \frac{\pi}{2}\right) = -\sin\theta$

**37.** $\sin^2\left(\theta - \frac{\pi}{2}\right) = \cos^2\theta$

**Solve each trigonometric equation for $0 \le \theta < 2\pi$.**

**38.** $\tan\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ $\frac{\pi}{2}$ and $\frac{3\pi}{2}$

**39.** $1 + \tan^2\theta = \cos\theta$ 0

---

### 4-7 Objectives

- To verify double-angle identities (p. 821)
- To verify half-angle identities (p. 822)

You can use double-angle and half-angle identities to find exact values of trigonometric expressions. In the half-angle identities, choose the positive or negative sign for each function depending on the quadrant in which $\frac{A}{2}$ lies.

**Double-angle identities**

$\cos 2\theta = \cos^2\theta - \sin^2\theta \qquad \sin 2\theta = 2\sin\theta\cos\theta \qquad \tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

$\cos 2\theta = 2\cos^2\theta - 1 \qquad \cos 2\theta = 1 - 2\sin^2\theta$

**Half-angle identities**

$\sin\frac{A}{2} = \pm\sqrt{\frac{1 - \cos A}{2}} \qquad \cos\frac{A}{2} = \pm\sqrt{\frac{1 + \cos A}{2}} \qquad \tan\frac{A}{2} = \pm\sqrt{\frac{1 - \cos A}{1 + \cos A}}$

**Use a double-angle identity to find the exact value of each expression.**

**40.** $\sin 120°$ $\frac{\sqrt{3}}{2}$

**41.** $\cos 90°$ 0

**42.** $\tan 300°$ $-\sqrt{3}$

**Use a half-angle identity to find the exact value of each expression.**

**43.** $\cos 180°$ $-1$

**44.** $\tan 60°$ $\sqrt{3}$

**45.** $\sin 120°$ $\frac{\sqrt{3}}{2}$



36. $\cos\left(\theta + \frac{\pi}{2}\right) = \cos\theta\cos\left(\frac{\pi}{2}\right) - \sin\theta\sin\left(\frac{\pi}{2}\right)$
$= \cos\theta \cdot 0 - \sin\theta \cdot 1$
$= -\sin\theta$

37. $\sin^2\left(\theta - \frac{\pi}{2}\right) = \sin^2\left(-\left(\frac{\pi}{2} - \theta\right)\right)$
$= \left(-\sin\left(\frac{\pi}{2} - \theta\right)\right) \cdot \left(-\sin\left(\frac{\pi}{2} - \theta\right)\right)$
$= (-\cos\theta)(-\cos\theta)$
$= \cos^2\theta$