Chapter Review

Vocabulary Review

asymptote (p. 433)	decay factor (p. 433)	logarithmic equation (p. 463)
Change of Base Formula (p. 461)	exponential equation (p. 461)	logarithmic function (p. 448)
common logarithm (p. 447)	exponential function (p. 430)	natural logarithmic function (p. 470)
continuously compounded interest formula (p. 441)	growth factor (p. 430)	
	logarithm (p. 447)	

Go Online
PHSchool.com
For: Vocabulary quiz
Web Code: agj-0851

Choose the correct term to complete each sentence.

1. In the exponential function $y = ab^x$, when $b > 1$, b is the __?__. **growth factor**
2. A __?__ is a logarithm that uses base 10. **common logarithm**
3. The line $x = 2$ is a(n) __?__ of the function $f(x) = \frac{2}{x-2}$. **asymptote**
4. The Change of Base Formula can be used to evaluate a __?__ with any base. **logarithm**
5. An __?__ can be solved by taking the logarithm of each side of the equation. **exponential equation**

Skills and Concepts

8-1 Objectives
- ▼ To model exponential growth (p. 430)
- ▼ To model exponential decay (p. 432)

The general form of an **exponential function** is $y = ab^x$, where x is a real number, $a \neq 0$, $b > 0$, and $b \neq 1$. When $b > 1$, the function models exponential growth, and b is the **growth factor**. When $0 < b < 1$, the function models exponential decay, and b is the **decay factor**.

Determine whether each equation represents exponential growth or exponential decay. Find the rate of increase or decrease for each model. Graph each equation.

6. $y = 5^x$ 7. $y = 2(4)^x$ 8. $y = 0.2(3.8)^x$ 9. $y = 3(0.25)^x$

6–9. See margin.

Write an exponential equation whose graph passes through the given points.

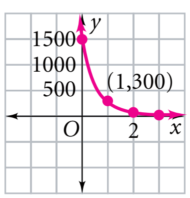
10. $(1, 5), (2, 7)$ 11. $(3, 1.5), (4, 15)$ 12. $\left(-1, 6\frac{3}{4}\right), \left(2, \frac{1}{4}\right)$ 13. $(-2, 9), (0, 1)$

10. $y = \frac{25}{7}\left(\frac{7}{5}\right)^x$
11. $y = 0.0015(10)^x$
12. $y = 2.25\left(\frac{1}{3}\right)^x$
13. $y = \left(\frac{1}{3}\right)^x$

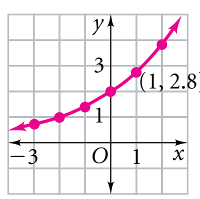
Write an exponential function to model each situation. Find the value of each function after five years, to the nearest dollar.

14. A $12,500 car depreciates 9% each year. **$y = 12{,}500(0.91)^x$; $7800**
15. A baseball card bought for $50 increases 3% in value each year. **$y = 50(1.03)^x$; $58**

Write an exponential equation for each graph. Evaluate the equation for $x = 4$.

16.


$y = 1500(0.2)^x$; 2.4

17.


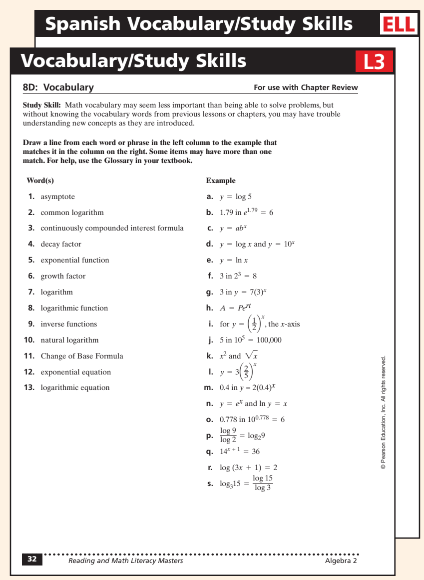
$y = 2(1.4)^x$; 7.6832

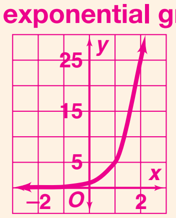
8 Chapter Review

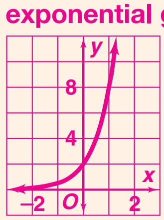
Resources

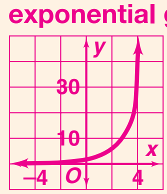
Student Edition
Extra Skills and Word Problems Practice, Ch. 8, p. 850
English/Spanish Glossary, p. 899
Properties and Formulas, p. 893
Table of Symbols, p. 889

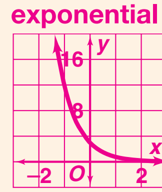
Differentiated Instruction
Vocabulary and Study Skills worksheet 8D
Spanish Vocabulary and Study Skills worksheet 8D
Interactive Textbook Audio Glossary
Online Vocabulary Quiz
Success Tracker™
Online at PHSchool.com



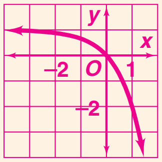
6. exponential growth; 400%


7. exponential growth; 300%


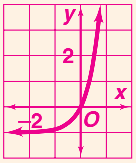
8. exponential growth; 280%


9. exponential decay; −75%


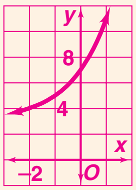
18. $y = 3^x$ reflected over the x-axis and translated up 1 unit



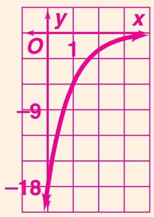
19. $y = 8^x$ translated down 1 unit



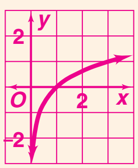
20. $y = 2(2)^x$ translated left 1 unit and up 3 units



21. $y = 2\left(\frac{1}{3}\right)^x$ reflected over the x-axis and translated right 2 units



40.



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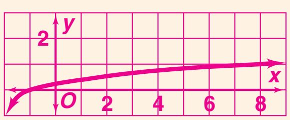
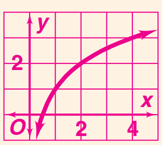
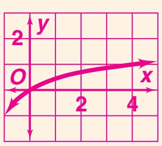


42.



43.



8-2 Objectives

- To identify the role of constants in $y = ab^{cx}$ (p. 439)
- To use e as a base (p. 441)

Exponential functions can be translated, stretched, shrunk, and reflected. The graph of $y = ab^{x-h} + k$ is the graph of $y = b^x$ stretched by a factor $|a|$, and translated $|h|$ units horizontally and $|k|$ units vertically.

The **continuously compounded interest** formula is $A = Pe^{rt}$, where P is the principal, r is the annual rate, and t is time in years.

Describe how the graph of each function relates to the graph of its parent function. Then graph each function. 18–21. See margin.

18. $y = -3^x + 1$ **19.** $y = 8^x - 1$

20. $y = 2(2)^{x+1} + 3$ **21.** $y = -2\left(\frac{1}{3}\right)^{x-2}$

Find the amount in a continuously compounded account for the given conditions.

22. principal: \$1000, annual interest rate: 4.8%, time: 2 yr **\$1100.76**

23. principal: \$250, annual interest rate: 6.2%, time: 2.5 yr **\$291.91**

24. principal: \$500, annual interest rate: 8.5%, time: 3 yr **\$645.23**

Evaluate to four decimal places.

25. e^1 **2.7183** **26.** e^{-1} **0.3679** **27.** e^5 **148.4132** **28.** $e^{-\frac{1}{2}}$ **0.6065**

29. **Physics** Radium has a half-life of 1620 years. Write the decay function for a 3-mg sample. Find the amount of radium remaining after 50 years. $y = 3\left(\frac{1}{2}\right)^{\frac{x}{1620}}$; ≈2.937 mg

8-3 Objectives

- To write and evaluate logarithmic expressions (p. 446)
- To graph logarithmic functions (p. 448)

If $y = b^x$, then $\log_b y = x$. The **logarithmic function** is the inverse of the exponential function, so the graphs of the functions are reflections of one another over the line $y = x$. Logarithmic functions can be translated, stretched, shrunk, and reflected.

When $b = 10$, the logarithm is called a **common logarithm**, which you can write as $\log y$.

30. **Chemistry** The pH of a substance equals $-\log[H^+]$, where $[H^+]$ is the concentration of hydrogen ions. A sample of well water has a pH of 5.7. Find the concentration of hydrogen ions in the sample. 2.0×10^{-6}

31. The concentration of hydrogen ions in water is 10^{-7}. Find the pH of water. **7**

Write each equation in logarithmic form.

32. $6^2 = 36$ $2 = \log_6 36$ **33.** $2^{-3} = 0.125$ $-3 = \log_2 0.125$

34. $3^3 = 27$ $3 = \log_3 27$ **35.** $10^{-3} = 0.001$ $-3 = \log 0.001$

Evaluate each logarithm.

36. $\log_2 64$ **6** **37.** $\log_3 \frac{1}{9}$ **−2** **38.** $\log 0.00001$ **−5** **39.** $\log_2 1$ **0**

Graph each logarithmic function. 40–43. See margin.

40. $y = \log_3 x$ **41.** $y = \log (x + 2)$

42. $y = \log_2 2x$ **43.** $y = \log_5 (x + 1)$

8-4 Objective

▼ To use the properties of logarithms (p. 454)

For any positive numbers, M, N, and b, $b \neq 1$, each of the following statements is true. Each can be used to rewrite a logarithmic expression.

- $\log_b MN = \log_b M + \log_b N$, by the Product Property
- $\log_b \frac{M}{N} = \log_b M - \log_b N$, by the Quotient Property
- $\log_b M^x = x \log_b M$, by the Power Property

Write each logarithmic expression as a single logarithm.

44. $\log 8 + \log 3$ **$\log 24$**

45. $\log_2 5 - \log_2 3$ **$\log_2 \frac{5}{3}$**

46. $4 \log_3 x + \log_3 7$ **$\log_3 7x^4$**

47. $\log z - \log y$ **$\log \frac{z}{y}$**

Expand each logarithm. State the properties of logarithms that you use.

48. $\log_4 x^2y^3$

49. $\log 4s^4t$

50. $\log_3 \frac{2}{x}$

51. $\log (x + 3)^2$

52. Use the formula $L = 10 \log \frac{I}{I_0}$. Suppose the sound intensity of a fan must be reduced by one third. By how many decibels would the loudness be decreased? **about 1.76 dB**

48. **$2 \log_4 x + 3 \log_4 y$; Product and Power Properties**

49. **$\log 4 + 4 \log S + \log t$; Product and Power Properties**

50. **$\log_3 2 - \log_3 x$; Quotient Property**

51. **$2 \log (x + 3)$; Power Property**

8-5 Objectives

▼ To solve exponential equations (p. 461)

▼ To solve logarithmic equations (p. 463)

An equation in the form $b^{cx} = a$, where the exponent includes a variable, is called an **exponential equation**. You can solve exponential equations by taking the logarithm of each side of the equation. An equation that includes a logarithmic expression is called a **logarithmic equation**.

Solve each equation. Round your answers to the nearest hundredth.

53. $4^x = 27$ **2.38**

54. $3^x = 36$ **3.26**

55. $7^{x-3} = 25$ **4.65**

56. $5^x = 9$ **1.37**

Solve by graphing.

57. $5^{2x} = 25$ **1**

58. $3^{7x} = 160$ **≈0.66**

59. $6^{3x+1} = 215$ **≈0.67**

60. $0.5^x = 0.12$ **≈3.06**

Solve each logarithmic equation. Leave your answer in exact form.

61. $\log 3x = 1$ **$\frac{10}{3}$**

62. $\log_2 4x = 5$ **8**

63. $\log x = \log 2x^2 - 2$ **50**

64. $2 \log_3 x = 54$ **7,625,597,484,987**

65. Convert $\log_2 7$ to a logarithm in base 5. **$\log_5 91.68$**

66. **Biology** A culture of 10 bacteria is started, and the number of bacteria will double every hour. In about how many hours will there be 3,000,000 bacteria? **about 18.2 h**

8-6 Objectives

▼ To evaluate natural logarithmic expressions (p. 470)

▼ To solve equations using natural logarithms (p. 471)

The inverse of $y = e^x$ is the **natural logarithmic function** $y = \log_e x = \ln x$. You solve natural logarithm equations in the same way as common logarithm equations.

Solve each equation.

67. $e^{3x} = 12$ **0.83**

68. $\ln x + \ln(x + 1) = 2$ **2.26**

69. $2 \ln x + 3 \ln 2 = 5$ **4.31**

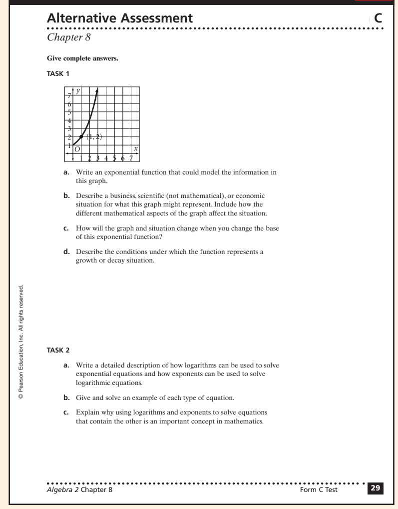
70. $\ln 4 - \ln x = 10$ **0.00018**

71. $4e^{(x-1)} = 64$ **3.77**

72. $3 \ln x + \ln 5 = 7$ **6.03**

73. **Savings** An initial investment of $350 is worth $429.20 after six years of continuous compounding. Find the interest rate. **about 3.4%**

Alternative Assessment L4


Alternative Assessment C
Chapter 8

Give complete answers.

TASK 1

a. Write an exponential function that could model the information in this graph.

b. Describe a business, scientific (not mathematical), or economic situation for what this graph might represent. Include how the different mathematical aspects of the graph affect the situation.

c. How will the graph and situation change when you change the base of this exponential function?

d. Describe the conditions under which the function represents a growth or decay situation.

TASK 2

a. Write a detailed description of how logarithms can be used to solve exponential equations and how exponents can be used to solve logarithmic equations.

b. Give and solve an example of each type of equation.

c. Explain why using logarithms and exponents to solve equations that contain the other is an important concept in mathematics.

Algebra 2 Chapter 8 Form C Test 29